

Student handout

1 1-D Particle-in-a-Box

Hamiltonian:

$$\hat{H} \doteq -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \quad (1)$$

Eigenstates:

$$|n\rangle \doteq \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \quad (2)$$

$$n = \{1, 2, 3, \dots\} \quad (3)$$

Eigenvalue Equations:

$$\hat{H} |n\rangle = \frac{\pi^2 \hbar^2}{2\mu L^2} n^2 |n\rangle \quad (4)$$

$$(5)$$

2 Particle-on-a-Ring

Hamiltonian:

$$\hat{H} \doteq -\frac{\hbar^2}{2I} \frac{\partial^2}{\partial \phi^2} \quad (6)$$

Eigenstates:

$$|m\rangle \doteq \frac{1}{\sqrt{2\pi r_0}} e^{im\phi} \quad (7)$$

$$m = \{\dots - 2, -1, 0, 1, 2, \dots\} \quad (8)$$

Eigenvalue Equations:

$$\hat{H} |m\rangle = \frac{\hbar^2}{2I} m^2 |m\rangle \quad (9)$$

$$\hat{L}^2 |m\rangle = \hbar^2 m^2 |m\rangle \quad (10)$$

$$\hat{L}_z |m\rangle = \hbar m |m\rangle \quad (11)$$

3 2-D Particle-in-a-Box

Hamiltonian:

$$\hat{H} \doteq -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \quad (12)$$

Eigenstates:

$$|mn\rangle \doteq \sqrt{\frac{2}{L_x}} \sqrt{\frac{2}{L_y}} \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \quad (13)$$

$$m = \{1, 2, 3, \dots\} \quad (14)$$

$$n = \{1, 2, 3, \dots\} \quad (15)$$

Eigenvalue Equations:

$$\hat{H} |mn\rangle = \frac{\pi^2 \hbar^2}{2\mu} \left(\frac{m^2}{L_x^2} + \frac{n^2}{L_y^2} \right) |mn\rangle \quad (16)$$

$$(17)$$

4 Particle-on-a-Sphere

Hamiltonian:

$$\hat{H} \doteq -\frac{\hbar^2}{2I} \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right] \quad (18)$$

Eigenstates:

$$|\ell m\rangle \doteq Y_\ell^m(\theta, \phi) \quad (19)$$

$$= (-1)^{\frac{m+|m|}{2}} \sqrt{\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos \theta) e^{im\phi} \quad (20)$$

$$\ell = \{0, 1, 2, \dots\} \quad (21)$$

$$m = \{-\ell, \dots, 0, \dots, \ell\} \quad (22)$$

Eigenvalue Equations:

$$\hat{H} |\ell m\rangle = \frac{\hbar^2}{2I} \ell(\ell+1) |\ell m\rangle \quad (23)$$

$$\hat{L}^2 |\ell m\rangle = \hbar^2 \ell(\ell+1) |\ell m\rangle \quad (24)$$

$$\hat{L}_z |\ell m\rangle = \hbar m |\ell m\rangle \quad (25)$$

5 Hydrogen Atom

Hamiltonian:

$$\hat{H} \doteq -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{ke^2}{r} \quad (26)$$

$$\doteq -\frac{\hbar^2}{2\mu r^2} \left[\frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right] - \frac{ke^2}{r} \quad (27)$$

Eigenstates:

$$|n\ell m\rangle \doteq R_{n\ell}(r) Y_\ell^m(\theta, \phi) \quad (28)$$

$$= -\sqrt{\left(\frac{2Z}{na_0}\right)^3 \frac{(n-\ell-1)!}{2n[(n+\ell)!]^3} \left(\frac{2\rho}{n}\right)^\ell} e^{-\frac{\rho}{n}} L_{n+\ell}^{2\ell+1}\left(\frac{2\rho}{n}\right) (-1)^{\frac{m+|m|}{2}} \sqrt{\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos \theta) e^{im\phi} \quad (29)$$

$$\rho = \frac{Zr}{a_0} \quad (30)$$

$$n = \{1, 2, 3, \dots\} \quad (31)$$

$$\ell = \{0, 1, 2, \dots, n-1\} \quad (32)$$

$$m = \{-\ell, \dots, 0, \dots, \ell\} \quad (33)$$

Eigenvalue Equations:

$$\hat{H} |n\ell m\rangle = -\frac{1}{2} \left(\frac{Ze^2}{4\pi\epsilon_0} \right)^2 \frac{\mu}{\hbar^2} \frac{1}{n^2} |n\ell m\rangle \quad (34)$$

$$= -13.6\text{eV} \frac{1}{n^2} |n\ell m\rangle \quad (35)$$

$$\hat{L}^2 |n\ell m\rangle = \hbar^2 \ell(\ell+1) |n\ell m\rangle \quad (36)$$

$$\hat{L}_z |n\ell m\rangle = \hbar m |n\ell m\rangle \quad (37)$$