1 Diatomic hydrogen

At low temperatures, a diatomic molecule can be well described as a *rigid rotor*. The Hamiltonian of such a system is simply proportional to the square of the angular momentum

$$H = \frac{1}{2I} L^2 \tag{1}$$

and the energy eigenvalues are

$$E_{\ell m} = \hbar^2 \frac{\ell(\ell+1)}{2I} \tag{2}$$

(a) What is the energy of the ground state and the first and second excited states of the H_2 molecule? i.e. the lowest three distinct energy eigenvalues.

(b) At room temperature, what is the relative probability of finding a hydrogen molecule in the $\ell = 0$ state versus finding it in any one of the $\ell = 1$ states?
i.e. what is $P_{\ell=0,m=0} / \left(P_{\ell=1,m=-1} + P_{\ell=1,m=0} + P_{\ell=1,m=1}\right)$

(c) At what temperature is the value of this ratio 1?

(d) At room temperature, what is the probability of finding a hydrogen molecule in any one of the $\ell = 2$ states versus that of finding it in the ground state?
i.e. what is $P_{\ell=0,m=0} / \left(P_{\ell=2,m=-2} + P_{\ell=2,m=-1} + \cdots + P_{\ell=2,m=2}\right)$