1 Isolength and Isoforce Stretchability

In class, you measured the isolength stretchability and the isoforce stretchability of your systems in the PDM. We found that for some systems these were very different, while for others they were identical.

Show with algebra (NOT experiment) that the ratio of isolength stretchability to isoforce stretchability is the same for both the left-hand side of the system and the right-hand side of the system. i.e.:

$$\frac{\left(\frac{\partial x_L}{\partial F_L}\right)_{x_R}}{\left(\frac{\partial x_L}{\partial F_L}\right)_{F_R}} = \frac{\left(\frac{\partial x_R}{\partial F_R}\right)_{x_L}}{\left(\frac{\partial x_R}{\partial F_R}\right)_{F_L}} \tag{1}$$

Hint You will need to make use of the *cyclic chain rule*:

$$\left(\frac{\partial A}{\partial B}\right)_C = -\left(\frac{\partial A}{\partial C}\right)_B \left(\frac{\partial C}{\partial B}\right)_A \tag{2}$$

Hint You will also need the *ordinary chain rule*:

$$\left(\frac{\partial A}{\partial B}\right)_D = \left(\frac{\partial A}{\partial C}\right)_D \left(\frac{\partial C}{\partial B}\right)_D \tag{3}$$