1 Nucleus in a Magnetic Field

Nuclei of a particular isotope species contained in a crystal have spin $I = 1$, and thus, $m = \{+1, 0, -1\}$. The interaction between the nuclear quadrupole moment and the gradient of the crystalline electric field produces a situation where the nucleus has the same energy, $E = \varepsilon$, in the state $m = +1$ and the state $m = -1$, compared with an energy $E = 0$ in the state $m = 0$, i.e. each nucleus can be in one of 3 states, two of which have energy $E = \varepsilon$ and one has energy $E = 0$.

(a) Find the Helmholtz free energy $F = U - TS$ for a crystal containing N nuclei which do not interact with each other.

(b) Find an expression for the entropy as a function of temperature for this system. (Hint: use results of part a.)

(c) Indicate what your results predict for the entropy at the extremes of very high temperature and very low temperature.