1 Inhomogeneous Linear ODEs with Constant Coefficients (First Example)

Inhomogeneous, linear ODEs with constant coefficients are among the most straigtforward to solve, although the algebra can get messy. This content should have been covered in your Differential Equations course (MTH 256 or equiv.). If you need a review, please see: The Method for Inhomogeneous Equations or your differential equations text.

The general solution of the **homogeneous** differential equation

$$\ddot{x} - \dot{x} - 6x = 0$$

is

$$x(t) = A\, e^{3t} + B\, e^{-2t}$$

where A and B are arbitrary constants that would be determined by the initial conditions of the problem.

(a) Find a particular solution of the **in**homogeneous differential equation $\ddot{x} - \dot{x} - 6x = -25\sin(4t)$.

(b) Find the general solution of $\ddot{x} - \dot{x} - 6x = -25\sin(4t)$.

(c) Some terms in your general solution have an undetermined coefficients, while some coefficients are fully determined. Explain what is different about these two cases.

(d) Find a particular solution of $\ddot{x} - \dot{x} - 6x = 12e^{-3t}$

(e) Find the general solution of $\ddot{x} - \dot{x} - 6x = 12e^{-3t} - 25\sin(4t)$

How is this general solution related to the particular solutions you found in the previous parts of this question?

Can you add these particular solutions together with arbitrary coefficients to get a new particular solution?

(f) Sense-making: **Check your answer;** Explicitly plug in your final answer in part (e) and check that it satisfies the differential equation.