# 1 Diagonalization

(a) Let

$$|\alpha\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \text{and} \qquad |\beta\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

Show that $|\alpha\rangle$ and $|\beta\rangle$ are orthonormal. (If a pair of vectors is orthonormal, that suggests that they might make a good basis.)

(b) Consider the matrix

$$C \doteq \begin{pmatrix}3 & 1\\1 & 3\end{pmatrix}$$

Show that the vectors $|\alpha\rangle$ and $|\beta\rangle$ are eigenvectors of C and find the eigenvalues. (Note that showing something is an eigenvector of an operator is far easier than finding the eigenvectors if you don't know them!)

(c) A operator is always represented by a diagonal matrix if it is written in terms of the basis of its own eigenvectors. What does this mean? Find the matrix elements for a new matrix $E$ that corresponds to $C$ expanded in the basis of its eigenvectors, i.e. calculate $\langle\alpha|C|\alpha\rangle$, $\langle\alpha|C|\beta\rangle$, $\langle\beta|C|\alpha\rangle$ and $\langle\beta|C|\beta\rangle$ and arrange them into a sensible matrix $E$. Explain why you arranged the matrix elements in the order that you did.

(d) Find the determinants of $C$ and $E$. How do these determinants compare to the eigenvalues of these matrices?