1 Eigen Spin Challenge

Consider the arbitrary Pauli matrix $\sigma_n = \hat{n} \cdot \vec{\sigma}$ where $\hat{n}$ is the unit vector pointing in an arbitrary direction.

(a) Find the eigenvalues and normalized eigenvectors for σ_n. The answer is:

$$\begin{pmatrix} \cos\frac{\theta}{2} e^{-i\phi/2} \\ \sin\frac{\theta}{2} e^{i\phi/2} \end{pmatrix} \begin{pmatrix} -\sin\frac{\theta}{2} e^{-i\phi/2} \\ \cos\frac{\theta}{2} e^{i\phi/2} \end{pmatrix}$$

It is not sufficient to show that this answer is correct by plugging into the eigenvalue equation. Rather, you should do all the steps of finding the eigenvalues and eigenvectors as if you don't know the answer. Hint: $\sin\theta = \sqrt{1 - \cos^2\theta}$.

(b) Show that the eigenvectors from part (a) above are orthogonal.

(c) Simplify your results from part (a) above by considering the three separate special cases: $\hat{n} = \hat{\imath}$, $\hat{n} = \hat{\jmath}$, $\hat{n} = \hat{k}$. In this way, find the eigenvectors and eigenvalues of σ_x, σ_y, and σ_z.