

1 Spin Matrix

The Pauli spin matrices σ_x , σ_y , and σ_z are defined by:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

These matrices are related to angular momentum in quantum mechanics.

- (a) By drawing pictures, convince yourself that the arbitrary unit vector \hat{n} can be written as:

$$\hat{n} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z}$$

where θ and ϕ are the parameters used to describe spherical coordinates.

- (b) Find the entries of the matrix $\hat{n} \cdot \vec{\sigma}$ where the “matrix-valued-vector” $\vec{\sigma}$ is given in terms of the Pauli spin matrices by

$$\vec{\sigma} = \sigma_x \hat{x} + \sigma_y \hat{y} + \sigma_z \hat{z}$$

and \hat{n} is given in part (a) above.