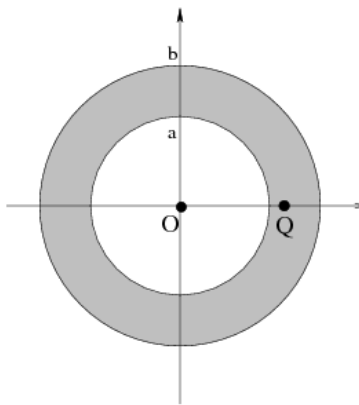


# 1 Gravitational Field and Mass

The gravitational field due to a spherical shell of matter (or equivalently, the electric field due to a spherical shell of charge) is given by:

$$\vec{g} = \begin{cases} 0 & \text{for } r < a \\ -G \frac{M}{b^3 - a^3} \left( r - \frac{a^3}{r^2} \right) \hat{r} & \text{for } a < r < b \\ -G \frac{M}{r^2} \hat{r} & \text{for } r > b \end{cases} \quad (1)$$



This problem explores the consequences of the divergence theorem for this shell.

- (a) Using the given description of the gravitational field, find the divergence of the gravitational field everywhere in space. You will need to divide this question up into three parts:  $r < a$ ,  $a < r < b$ , and  $r > b$ .
- (b) Briefly discuss the physical meaning of the divergence in this particular example.
- (c) For this gravitational field, verify the divergence theorem on a sphere, concentric with the shell, with radius  $Q$ , where  $a < Q < b$ . ("Verify" the divergence theorem means calculate the integrals from both sides of the divergence theorem and show that they give the same answer.)
- (d) Briefly discuss how this example would change if you were discussing the electric field of a uniformly charged spherical shell.