

1 Quantum Particle in a 2-D Box

You know that the normalized spatial eigenfunctions for a particle in a 1-D box of length L are $\sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$. If you want the eigenfunctions for a particle in a 2-D box, then you just *multiply* together the eigenfunctions for a 1-D box in each direction. (This is what the separation of variables procedure tells you to do.)

- Find the normalized eigenfunctions for a particle in a 2-D box with sides of length L in the x -direction and length W in the y -direction.
- Find the Hamiltonian for a 2-D box and show that your eigenstates are indeed eigenstates and find a formula for the possible energies
- Any sufficiently smooth spatial wave function inside a 2-D box can be expanded in a **double** sum of the product wave functions, i.e.

$$\psi(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_{nm} \text{eigenfunction}_n(x) \text{eigenfunction}_m(y) \quad (1)$$

Using your expressions from part (a) above, write out all the terms in this sum out to $n = 3$, $m = 3$. Arrange the terms, conventionally, in terms of increasing energy.

You may find it easier to work in bra/ket notation:

$$\begin{aligned} |\psi\rangle &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_{nm} |n\rangle |m\rangle \\ &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_{nm} |nm\rangle \end{aligned}$$

- Find a formula for the c_{nm} s in part (c). Find the formula first in bra ket notation and then rewrite it in wave function notation.