

1 Completeness Relation Change of Basis

(a) Given the polar basis kets written as a superposition of Cartesian kets

$$\begin{aligned} |\hat{s}\rangle &= \cos \phi |\hat{x}\rangle + \sin \phi |\hat{y}\rangle \\ |\hat{\phi}\rangle &= -\sin \phi |\hat{x}\rangle + \cos \phi |\hat{y}\rangle \end{aligned}$$

Find the following quantities:

$$\langle \hat{x} | \hat{s} \rangle, \quad \langle \hat{y} | \hat{s} \rangle, \quad \langle \hat{x} | \hat{\phi} \rangle, \quad \langle \hat{y} | \hat{\phi} \rangle$$

(b) Given a vector written in the polar basis

$$|\vec{v}\rangle = a |\hat{s}\rangle + b |\hat{\phi}\rangle$$

where a and b are known.

Express $|\vec{v}\rangle$ in the Cartesian basis,

$$|\vec{v}\rangle = c |\hat{x}\rangle + d |\hat{y}\rangle$$

by finding c and d

Hint: Use the completeness relation: $|\hat{x}\rangle \langle \hat{x}| + |\hat{y}\rangle \langle \hat{y}| = 1$

(c) Given a quantum state written in the S_z basis,

$$|\Psi\rangle = g |+\rangle + h |-\rangle,$$

express $|\Psi\rangle$ in the S_y basis. That is, find coefficients j and k such that

$$|\Psi\rangle = j |+\rangle_y + k |-\rangle_y.$$