1 Coefficients in Wavefunction notation

For each of the following, use the completeness relation of the position eigenstates to show:

(a) that the state of the system can be expressed as $|\psi\rangle = \int \psi(x)\,|x\rangle\,dx$

(b) that the expansion coeffients for a state written in the energy basis, $c_n = \langle n|\psi\rangle$ are $\int \varphi_n^*(x)\psi(x)dx$ in wavefunction notation.

(c) that the norm of a quantum state, $\langle\psi|\psi\rangle$ is $\int \psi^*(x)\psi(x)dx$ in wavefunction notation.

(d) that the expectation value of position,

$$\langle\hat{X}\rangle = \langle\psi|\,\hat{X}\,|\psi\rangle = \int |\psi(x)|^2 x\;dx$$

in wavefunction notation.