1 Find Force Law: Logarithmic Spiral Orbit

(Use the equation for orbit shape.) *Gain experience with unusual force laws.*

In science fiction movies, characters often talk about a spaceship "spiralling in" right before it hits the planet. But all orbits in a $1/r^2$ force are conic sections, not spirals. This spiralling in happens because the spaceship hits atmosphere and the drag from the atmosphere changes the shape of the orbit. But, in an alternate universe, we might have other force laws.

In class, we discussed how to calculate the shape of the orbit for an inverse square potential. More generally, the equation for the orbit of a mass μ under the influence of a central force $f(r)$ is given by:

$$\frac{d^2u}{d\phi^2} + u = -\frac{\mu}{\ell^2}\frac{1}{u^2} f\left(\frac{1}{u}\right) \tag{1}$$

$$\Rightarrow f\left(\frac{1}{u}\right) = -\frac{\ell^2}{\mu} u^2 \left(\frac{d^2u}{d\phi^2} + u\right) \tag{2}$$

where $u = r^{-1}$.

Find the force law for a mass μ, under the influence of a central-force field, that moves in a logarithmic spiral orbit given by $r = ke^{\alpha\phi}$, where k and α are constants.