1 Inner Product Properties

The properties that an inner product on an abstract vector space must satisfy can be found in: Definition and Properties of an Inner Product. Definition: The inner product for any two vectors in the vector space of periodic functions with a given period (let's pick 2π for simplicity) is given by:

$$\langle f|g\rangle = \int_0^{2\pi} f^*(x)\, g(x)\, dx$$

(a) Show that the first property of inner products

$$\langle f|g\rangle = \langle g|f\rangle^*$$

is satisfied for this definition.

(b) Show that the second property of inner products

$$\langle f|\left(\lambda\,|g\rangle + \mu\,|h\rangle\right) = \lambda\,\langle f|g\rangle + \mu\,\langle f|h\rangle$$

is satisfied for this definition.