1 Approximating a Delta Function with Isoceles Triangles

The delta function is defined so that

$$\delta(x-a) = \begin{cases} 0, & x \neq a \\ \infty, & x = a \end{cases}$$

and normalized so that

$$\int_{-\infty}^{\infty} \delta(x-a)\,dx = 1$$

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(a) Find a set of functions that approximate the delta function $\delta(x-a)$ with a sequence of isosceles triangles $\delta_\epsilon(x-a)$, centered at a, that get narrower and taller as the parameter ϵ approaches zero. Choose the approximation functions so that their integral over all space is always equal to one.

(b) Using the test function $f(x) = 3x^2$, find the value of

$$\int_{-\infty}^{\infty} f(x)\delta_\epsilon(x-a)\,dx$$

Then, show that the integral approaches $f(a)$ in the limit that $\epsilon \to 0$.