1 Coffees and Bagels and Net Worth

In economics, the term *utility* is roughly related to overall happiness. Many things affect your happiness, including the amount of money you have and the amount of coffee you drink. We cannot directly measure your happiness, but we *can* measure how much money you are willing to give up in order to obtain coffee or bagels. If we assume you choose wisely, we can thus determine that your happiness increases when you decrease your amount of money by that amount in exchange for increasing your coffee consumption. Thus money is a (poor) measure of happiness or utility.

Money is also a nice quantity because it is conserved—just like energy! You may gain or lose money, but you always do so by a transaction. (There are some exceptions to the conservation of money, but they involve either the Fed, counterfeiters, or destruction of cash money, and we will ignore those issues.)

In this problem, we will assume that you have bought all the coffee and bagels you want (and no more), so that your happiness has been maximized. Thus you are in equilibrium with the coffee shop. We will assume further that you remain in equilibrium with the coffee shop at all times, and that you can sell coffee and bagels back to the coffee shop at cost.[1]

Thus your savings S can be considered to be a function of your bagels B and coffee C. In this problem we will also discuss the prices P_B and P_C, which you may *not* assume are independent of B and C. It may help to imagine that you could possibly buy out the local supply of coffee, and have to import it at higher costs.

(a) The prices of bagels and coffee P_B and P_C have derivative relationships between your savings and the quantity of coffee and bagels that you have. What are the units of these prices? What is the mathematical definition of P_C and P_B?

(b) Write down the total differential of your savings, in terms of B, C, P_B and P_C.

(c) Solve for the total differential of your net worth. Your net worth W is the sum of your total savings plus the value of the coffee and bagels that you own. From the total differential, relate your amount of coffee and bagels to partial derivatives of your net worth.

[1] Yes, this is ridiculous. It would be slightly less ridiculous if we were talking about nations and commodities, but also far less humorous.