1 Quantum harmonic oscillator

(a) Find the entropy of a set of N oscillators of frequency ω as a function of the total quantum number n. Use the multiplicity function:

$$g(N, n) = \frac{(N+n-1)!}{n!(N-1)!} \tag{1}$$

and assume that $N \gg 1$. This means you can make the Sitrling approximation that $\log N! \approx N \log N - N$. It also means that $N - 1 \approx N$.

(b) Let U denote the total energy $n\hbar\omega$ of the oscillators. Express the entropy as $S(U, N)$. Show that the total energy at temperature T is

$$U = \frac{N\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1} \tag{2}$$

This is the Planck result found the **hard** way. We will get to the easy way soon, and you will never again need to work with a multiplicity function like this.