1 Working with Representations on the Ring

The following are 3 different representations for the **same** state on a quantum ring for $r_0 = 1$

$$|\Phi_a\rangle = i\sqrt{\frac{2}{12}}\,|3\rangle - \sqrt{\frac{1}{12}}\,|1\rangle + \sqrt{\frac{3}{12}}e^{i\frac{\pi}{4}}\,|0\rangle - i\sqrt{\frac{2}{12}}\,|-1\rangle + \sqrt{\frac{4}{12}}\,|-3\rangle \quad (1)$$

$$|\Phi_b\rangle \doteq \begin{pmatrix} \vdots \\ i\sqrt{\frac{2}{12}} \\ 0 \\ -\sqrt{\frac{1}{12}} \\ \sqrt{\frac{3}{12}}e^{i\frac{\pi}{4}} \\ -i\sqrt{\frac{2}{12}} \\ 0 \\ \sqrt{\frac{4}{12}} \\ \vdots \end{pmatrix} \begin{matrix} \\ \\ \\ \\ \leftarrow m = 0 \\ \\ \\ \\ \\ \end{matrix} \quad (2)$$

$$\Phi_c(\phi) \doteq \sqrt{\frac{1}{24\pi}}\left(i\sqrt{2}e^{i3\phi} - e^{i\phi} + \sqrt{3}e^{i\frac{\pi}{4}} - i\sqrt{2}e^{-i\phi} + \sqrt{4}e^{-i3\phi}\right) \quad (3)$$

(a) With each representation of the state given above, explicitly calculate the probability that $L_z = -1\hbar$. Then, calculate all other non-zero probabilities for values of L_z with a method/representation of your choice.

(b) Explain how you could be sure you calculated all of the non-zero probabilities.

(c) If you measured the z-component of angular momentum to be $3\hbar$, what would the state of the particle be immediately after the measurement is made?

(d) With each representation of the state given above, explicitly calculate the probability that $E = \frac{9}{2}\frac{\hbar^2}{I}$. Then, calculate all other non-zero probabilities for values of E with a method of your choice.

(e) If you measured the energy of the state to be $\frac{9}{2}\frac{\hbar^2}{I}$, what would the state of the particle be immediately after the measurement is made?