1 Paramagnetism

Find the equilibrium value at temperature T of the fractional magnetization

$$\frac{\mu_{tot}}{Nm} \equiv \frac{2\langle s \rangle}{N} \tag{1}$$

of a system of N spins each of magnetic moment m in a magnetic field B. The spin excess is $2s$. The energy of this system is given by

$$U = -\mu_{tot} B \tag{2}$$

where μ_{tot} is the total magnetization. Take the entropy as the logarithm of the multiplicity $g(N, s)$ as given in (1.35 in the text):

$$S(s) \approx k_B \log g(N, 0) - k_B \frac{2s^2}{N} \tag{3}$$

for $|s| \ll N$, where s is the spin excess, which is related to the magnetization by $\mu_{tot} = 2sm$. *Hint*: Show that in this approximation

$$S(U) = S_0 - k_B \frac{U^2}{2m^2 B^2 N}, \tag{4}$$

with $S_0 = k_B \log g(N, 0)$. Further, show that $\frac{1}{kT} = -\frac{U}{m^2 B^2 N}$, where U denotes $\langle U \rangle$, the thermal average energy.