1 Mass-radius relationship for white dwarfs

Consider a white dwarf of mass M and radius R. The dwarf consists of ionized hydrogen, thus a bunch of free electrons and protons, each of which are fermions. Let the electrons be degenerate but nonrelativistic; the protons are nondegenerate.

(a) Show that the order of magnitude of the gravitational self-energy is $-\frac{GM^2}{R}$, where G is the gravitational constant. (If the mass density is constant within the sphere of radius R, the exact potential energy is $-\frac{5}{3}\frac{GM^2}{R}$).

(b) Show that the order of magnitude of the kinetic energy of the electrons in the ground state is

$$\frac{\hbar^2 N^{\frac{5}{3}}}{mR^2} \approx \frac{\hbar^2 M^{\frac{5}{3}}}{m M_H^{\frac{5}{3}} R^2} \tag{1}$$

where m is the mass of an electron and M_H is the mas of a proton.

(c) Show that if the gravitational and kinetic energies are of the same order of magnitude (as required by the virial theorem of mechanics), $M^{\frac{1}{3}} R \approx 10^{20}\mathrm{g}^{\frac{1}{3}}\mathrm{cm}$.

(d) If the mass is equal to that of the Sun ($2 \times 10^{33} g$), what is the density of the white dwarf?

(e) It is believed that pulsars are stars composed of a cold degenerate gas of neutrons (i.e. neutron stars). Show that for a neutron star $M^{\frac{1}{3}} R \approx 10^{17}\mathrm{g}^{\frac{1}{3}}\mathrm{cm}$. What is the value of the radius for a neutron star with a mass equal to that of the Sun? Express the result in km.