

1 Photon carnot engine

In our week on radiation, we saw that the Helmholtz free energy of a box of radiation at temperature T is

$$F = -8\pi \frac{V(kT)^4 \pi^4}{h^3 c^3 45} \quad (1)$$

From this we also found the internal energy and entropy

$$U = 24\pi \frac{(kT)^4 \pi^4}{h^3 c^3 45} V \quad (2)$$

$$S = 32\pi kV \left(\frac{kT}{hc} \right)^3 \frac{\pi^4}{45} \quad (3)$$

Given these results, let us consider a Carnot engine that uses an empty metallic piston (i.e. a photon gas).

- (a) Given T_H and T_C , as well as V_1 and V_2 (the two volumes at T_H), determine V_3 and V_4 (the two volumes at T_C).
- (b) What is the heat Q_H taken up and the work done by the gas during the first isothermal expansion? Are they equal to each other, as for the ideal gas?
- (c) Does the work done on the two isentropic stages cancel each other, as for the ideal gas?
- (d) Calculate the total work done by the gas during one cycle. Compare it with the heat taken up at T_H and show that the energy conversion efficiency is the Carnot efficiency.