

Submit these problems on Gradescope by 3 pm on Monday 14 October.

1 Tetrahedron

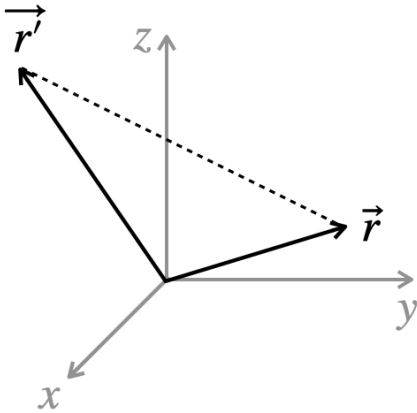
Using a dot product, find the angle between any two line segments that join the center of a regular tetrahedron to its vertices. *Hint:* Think of the vertices of the tetrahedron as sitting at the vertices of a cube (at coordinates $(0,0,0)$, $(1,1,0)$, $(1,0,1)$ and $(0,1,1)$ —you may need to build a model and play with it to see how this works!)

2 Distance Formula in Curvilinear Coordinates

You might want to wait until after Friday's class to do this problem.

The distance $|\vec{r} - \vec{r}'|$ between the point $\vec{r}$ and the point $\vec{r}'$ is a coordinate-independent, physical and geometric quantity. But, in practice, you will need to know how to express this quantity in different coordinate systems.

- (a) Find the distance $|\vec{r} - \vec{r}'|$ between the point $\vec{r}$ and the point $\vec{r}'$ in rectangular coordinates.



- (b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r} - \vec{r}'| = \sqrt{s^2 + s'^2 - 2ss' \cos(\phi - \phi') + (z - z')^2}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

- (c) Show that this same distance written in spherical coordinates is:

$$|\vec{r} - \vec{r}'| = \sqrt{r'^2 + r^2 - 2rr' [\sin \theta \sin \theta' \cos(\phi - \phi') + \cos \theta \cos \theta']}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

- (d) Now assume that $\vec{r}'$ and $\vec{r}$ are in the x - y plane. Simplify the previous two formulas.

3 Linear Quadrupole (w/ series)

You might want to wait until after Friday's class to do this problem.

Consider a collection of three charges arranged in a line along the z -axis: charges $+Q$ at $z = \pm D$ and charge $-2Q$ at $z = 0$.

- (a) Find the electrostatic potential at a point $\vec{r}$ in the xy -plane at a distance s from the center of the quadrupole. The formula for the electrostatic potential V at a point $\vec{r}$ due to a charge Q at the point $\vec{r}'$ is given by:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{|\vec{r} - \vec{r}'|}$$

- (b) Assume $s \gg D$. Find the first two non-zero terms of a power series expansion to the electrostatic potential you found in the first part of this problem.
- (c) A series of charges arranged in this way is called a linear quadrupole. Why?