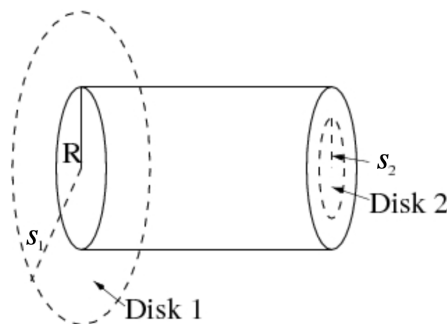


# 1 Ampere's Law for a Cylinder

(2, 2, 2, 2 pts)

In this problem, you will be investigating a cylindrical wire of finite radius  $R$ , carrying a non-uniform current density  $J = \kappa s \hat{z}$ , where  $\kappa$  is a constant and  $s$  is the distance from the axis of the cylinder.

- Find the total current flowing through the wire.
- Find the current flowing through Disk 2, a central (circular cross-section) portion of the wire out to a radius  $s_2 < R$ .



- Use Ampere's law to find the magnetic field at a distance  $s_1$  outside the wire.
- Use Ampere's law to find the magnetic field at a distance  $s_2$  inside the wire.

# 2 Magnetic Field and Current

(2, 4, 2 pts) Consider the magnetic field

$$\vec{B}(s, \phi, z) = \begin{cases} 0 & 0 \leq s < a \\ \alpha \frac{1}{s} (s^4 - a^4) \hat{\phi} & a < s < b \\ 0 & s > b \end{cases}$$

- Use step and/or delta functions to write this magnetic field as a single expression valid everywhere in space.
- Find a formula for the current density that creates this magnetic field.
- Interpret your formula for the current density, i.e. explain briefly in words where the current is.

### 3 Path Independence

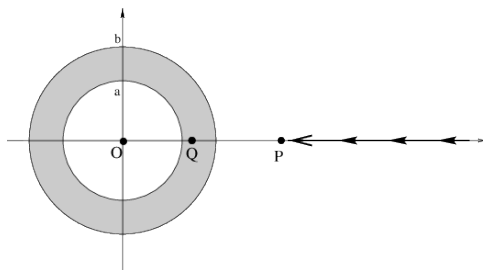
(2, 2, 2, 4, 2 pts)

The gravitational field due to a spherical shell of mass is given by:

$$\vec{g} = \begin{cases} 0 & r < a \\ -\frac{4}{3}\pi\rho G \left( r - \frac{a^3}{r^2} \right) \hat{r} & a < r < b \\ -\frac{4}{3}\pi\rho G \left( \frac{b^3 - a^3}{r^2} \right) \hat{r} & b < r \end{cases} \quad (1)$$

where  $a$  is the inside radius of the shell,  $b$  is the outside radius of the shell, and  $\rho$  is the constant mass density.

- (a) Using an explicit line integral, calculate the work required to bring a test mass, of mass  $m_0$ , from infinity to a point  $P$ , which is a distance  $c$  (where  $c > b$ ) from the center of the shell.



- (b) Using an explicit line integral, calculate the work required to bring the test mass along the same path, from infinity to the point  $Q$  a distance  $d$  (where  $a < d < b$ ) from the center of the shell.
- (c) Using an explicit line integral, calculate the work required to bring the test mass along the same radial path from infinity all the way to the center of the shell.
- (d) Using an explicit line integral, calculate the work required to bring in the test mass along the path drawn below, to the point  $P$  of the first question. Compare the work to your answer from the first question.
- (e) What is the work required to bring the test mass from infinity along the path drawn below to the point  $P$  of question a. Explain your reasoning.

