

## 1 Pipe flow line integral

For  $-3 \leq x \leq 3$  feet, the two-dimensional flow rate in feet per second of storm water in a pipe can be modeled by  $6e^{-x^2}\hat{\mathbf{y}}$ . For each of the following curves, choose a parametrization, construct a line integral of the given vector field over the curve and determine its (approximate) value.

- $x^2 + y^2 = 4$ .
- $y = 2 - x^2$ .
- $y = 2x$ .

## 2 Saturn and Titan

Titan is a moon of Saturn orbiting a distance  $1221.86 \times 10^3$  kilometers away and whose mass is  $2.36 \times 10^{-4}$  of Saturn's mass which is  $569 \times 10^{24}$  kilograms. The potential energy due to gravity is modeled by  $\Phi = \frac{-GMm}{r}$  where  $G = 6.67 \times 10^{-11} \frac{m^3 s^2}{kg}$ ,  $M$  is the mass of Saturn,  $m$  is the mass of an object (such as Titan) and  $r$  is the distance between Saturn and the object in orbit.

- Describe the level sets of  $\Phi$  where  $m$  and  $r$  are variables. On which level set is Titan?
- Which coordinate system would you choose for calculations in this situation?
- Determine the gradient of  $\Phi$ . In which direction does it point?