1 Gradient Practice

(2, 2, 2 pts)

Find the gradient of each of the following functions:

(a)

$$f(x, y, z) = e^{(x+y)} + x^2 y^3 \ln \frac{x}{z} \tag{1}$$

(b)

$$\sigma(\theta, \phi) = \cos\theta \sin^2 \phi \tag{2}$$

(c)

$$\rho(s, \phi, z) = (s + 3z)^2 \cos\phi \tag{3}$$

2 Linear Quadrupole (w/ series)

(4, 4, 2 pts)

Consider a collection of three charges arranged in a line along the z-axis: charges $+Q$ at $z = \pm D$ and charge $-2Q$ at $z = 0$.

(a) Find the electrostatic potential at a point $\vec{r}$ in the xy-plane at a distance s from the center of the quadrupole. The formula for the electrostatic potential V at a point $\vec{r}$ due to a charge Q at the point $\vec{r}'$ is given by:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{|\vec{r} - \vec{r}'|}$$

(b) Assume $s \gg D$. Find the first two non-zero terms of a power series expansion to the electrostatic potential you found in the first part of this problem.

(c) A series of charges arranged in this way is called a linear quadrupole. Why?