1 Entropy and Temperature

None Suppose the multiplicity of a system is $\Omega(U) = CU^{3N/2}$, where C is a constant and N is the number of particles.

(a) Show that $U = \frac{3}{2}Nk_BT$.

(b) Show that $\left(\frac{\partial^2 S}{\partial U^2}\right)_N$ is negative. This form of $\Omega(U)$ actually applies to a monatomic ideal gas.

2 Extensive Internal Energy

Consider a system which has an internal energy U defined by:

$$U = \gamma V^\alpha S^\beta \tag{1}$$

where α, β and γ are constants. The internal energy is an extensive quantity. What constraint does this place on the values α and β may have?

3 Rubber Sheet

Consider a hanging rectangular rubber sheet. We will consider there to be two ways to get energy into or out of this sheet: you can either stretch it vertically or horizontally. The vertical dimension of the rubber sheet we will call y, and the horizontal dimension of the rubber sheet we will call x. We can use these two independent variables to specify the "state" of the rubber sheet. Similiar to the partial derivative machine, we could choose any pair of variables from the set $\{x, y, F_x, F_y\}$ to specify the state of the rubber sheet.

If I pull the bottom down by a small distance Δy, with no horizontal force, what is the resulting change in width Δx? Express your answer in terms of partial derivatives of the potential energy $U(x, y)$.

4 Bottle in a Bottle

The internal energy of helium gas at temperature T is to a very good approximation given by

$$U = \frac{3}{2}Nk_BT \tag{2}$$

Consider a very irreversible process in which a small bottle of helium is placed inside a large bottle, which otherwise contains vacuum. The inner bottle contains a slow leak, so that the helium leaks into the outer bottle. The inner bottle contains one tenth the volume of the outer bottle, which is insulated. What is the change in temperature when this process is complete? What fraction of the helium will remain in the small bottle?