

# 1 Reflection and Transmission at a Sharp Boundary

A traveling wave in a string is incident from the left (L) and propagates at speed  $v_1$ , encountering a second string at  $x = 0$ . It partially reflects, returning in the same string with speed  $v_1$ . It partially continues into a second string, traveling with speed  $v_2$ .

$$\psi(x, t) = \begin{cases} \psi_L(x, t) = \text{Re}[Ae^{i(-\omega_1 t + k_1 x)}] + \text{Re}[Be^{i(-\omega_1 t - k_1 x)}] & \text{for } x \leq 0 \\ \psi_R(x, t) = \text{Re}[Ce^{i(-\omega_2 t + k_2 x)}] & \text{for } x \geq 0 \end{cases}$$

- Explain the meanings of the terms in the wavefunctions and say which directions the waves propagate.
- Explain why  $\omega_1 = \omega_2 = \omega$
- What equation represents the statement, “The string must be continuous at the boundary”? Show that it leads to  $A + B = C$ .
- Write the piecewise function for  $\frac{\partial \psi(x, t)}{\partial x}$ .
- What equation represents the statement, “The transverse component of the force at the boundary must sum to zero”? Show that it leads to  $k_1 A - k_1 B = k_2 C$
- Solve the equations in (c) and (e) and find the displacement reflection and transmission coefficients  $R_\psi \equiv \frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2}$  and  $T_\psi \equiv \frac{C}{A} = \frac{2k_1}{k_1 + k_2}$ .
- Look carefully at the expression in (d) and show that you can define a reflection coefficient for  $\frac{\partial \psi}{\partial x}$  and that it is  $R_{\frac{\partial \psi}{\partial x}} = \frac{k_2 - k_1}{k_1 + k_2}$
- Look carefully at the expression in (d) and show that you can define a transmission coefficient for  $\frac{\partial \psi}{\partial x}$  and that it is  $T_{\frac{\partial \psi}{\partial x}} = \frac{2k_2}{k_1 + k_2}$

# 2 Light Propagation in a Vacuum

*None* Light propagates in vacuum with speed  $c$  and in a medium with speed  $v = \frac{c}{n}$  where  $n$  is the refractive index ( $n > 1$ ). (The refractive index is not an integer!) Show that when light is incident from vacuum ( $n = 1$ ) onto glass ( $n = 1.5$ ), about 4% of the energy (which is proportional to  $|\psi|^2$ ) is reflected. Also show that the light changes its phase angle by  $\pi$  when it is reflected.

# 3 Square Wave in a Rope

*None* A rectangular traveling pulse is launched with speed  $v_1$  from the left into a very long rope. At the boundary at  $x = 0$ , the pulse is partially reflected and partially transmitted into a second rope where the transmitted pulse moves with velocity  $v_2$ . The reflected (black) and transmitted (red) pulses are depicted some time after the original pulse encounters the boundary. The system obeys the NDWE.

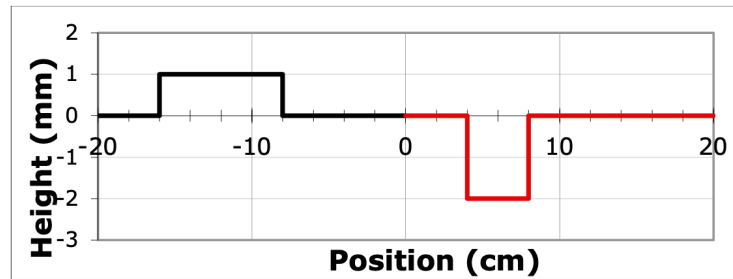


Figure: A point in time of the graph of a square wave after reflecting and transmitting at  $x = 0$ .

- (a) Use the widths of the reflected and transmitted pulses to find the ratio of  $v_1$  to  $v_2$ . Explain.
- (b) Calculate the relative velocities using the pulse locations and show that this is consistent with (a).
- (c) Is the mass density of the red rope smaller or larger than the black rope? Why?
- (d) Describe the original pulse (height, polarity, length), showing qualitative and quantitative reasoning.
- (e) Can you determine the velocities  $v_1$  and  $v_2$ ? If so, what are they, and if not, what information would you need?