

1 Commutator of Linear Transformations

Consider the following matrices:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Explain what each of the matrices “does” geometrically when thought of as a linear transformation acting on a vector.
- The commutator of two matrices A and B is defined by $[A, B] \stackrel{\text{def}}{=} AB - BA$. Find the following commutators: $[A, B]$, $[A, C]$, $[B, C]$. Two matrices are said to *commute*, if their commutator is zero.
- Thought of as linear transformations, two matrices commute if it doesn't matter in which order the transformations act. For all pairs of the matrices A , B , and C , discuss geometrically that the order of the transformations doesn't matter for the transformations that commute, but that the order does matter when the transformations don't commute.

2 Finding Orthogonal Vectors (Brief Version)

Consider the quantum state:

$$|\psi\rangle = \frac{1}{\sqrt{3}}|+\rangle + i\frac{\sqrt{2}}{\sqrt{3}}|-\rangle$$

Find the normalized vector $|\phi\rangle$ that is orthogonal to it.

3 Graphs of the Complex Conjugate

For each of the following complex numbers, determine the complex conjugate, square, and norm. Then, plot and clearly label each z , z^* , and $|z|$ on an Argand diagram.

- $z_1 = 4i - 3$
- $z_2 = 5e^{-i\pi/3}$
- $z_3 = -8$
- In a few full sentences, explain the geometric meaning of the complex conjugate and norm.

4 Representations of Complex Numbers—Table

Fill out the table above that asks you to do several simple complex number calculations in rectangular, polar, and exponential representations.

	Representation		
	Rectangular	Polar	Exponential
Z	$x + iy$	$r \cos \varphi + ir \sin \varphi$	$re^{i\varphi}$
Z^*			
$zz^* = z ^2$			
Z^2			