

1 Momentum of a Free Particle

Consider a free particle whose wave function is $\psi(x) = A \sin(p_0 x / \hbar)$,

- (a) Is this wave function an eigenstate of momentum?
- (b) What are the possible results of a measurement of the momentum?
- (c) Calculate the expectation value $\langle p \rangle$ and uncertainty Δp of momentum.

2 Dispersion Relation of a Free Particle

For a 1-D free particle, whose wave function is $\psi(x) = A e^{ikx}$, plot its dispersion relation, namely: the energy as a function of wave vector k . Note k can be positive or negative. (The dispersion relation will come back later in the course.)

3 Position and Momentum Commutation

Calculate the commutator of the position and momentum operators. Do this two ways:

- (a) using the position representation of the operators
- (b) using the momentum representation of the operators

4 Derivatives of the Gaussian

The normalized Gaussian function is of the form

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

- (a) Find the first two derivatives of the Gaussian function, by hand.
- (b) Make a table describing where the signs of the Gaussian itself and the signs of its first two derivatives are positive and negative.
- (c) Use your table to describe the shape of the Gaussian function.