1 Ground-State Uncertainty in an Infinite Potential Well

None Consider a particle confined in a one-dimensional infinite square potential well of width L, defined by the potential

$$V = \begin{cases} \infty & x < 0 \\ 0 & 0 \leq x \leq L \\ \infty & L < x \end{cases}$$

The normalized ground-state wavefunction of the particle is

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right), \qquad 0 \leq x \leq L.$$

Calcluate the postion uncertainty Δx and the momentum uncertainty Δp for the ground state of the particle, and confirm that $\Delta x \Delta p > \frac{\hbar}{2}$.

2 Reflection from a Square Barrier

Note for Liz: Take a look at the last part of this problem where they interpret when E=V0. I think it needs to be reformulated. Consider a particle with mass m and energy E incident from the left on a square potential barrier with height $V_0 > 0$:

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 < x < a \\ 0 & a < x \end{cases}$$

This is an example of an *unbounded* system, so there is no condition on the energy eigenvalue. There are two cases, $E > V_0$ and $E < V_0$. Consider only $E > V_0$.

(a) Set up a wave incident from the left, and one transmitted to the right, so the total wave function is:

$$\psi(x) = \begin{cases} e^{ik_1 x} + Ae^{-ik_1 x} & x < 0 \\ Ce^{ik_3 x} + De^{-ik_3 x} & 0 < x < a \\ Be^{ik_2 x} & a < x \end{cases}$$

Use the energy eigenvalue equation to solve for the values k_1, k_2, k_3 in terms of E and V_0.

(b) What are the boundary conditions that establish the relationship among the coefficients A, B, C, and D?

(c) The probability to observe the particle reflected is

$$r \equiv |A|^2$$

(remember we measure probabilities, and not amplitudes). Find r. Also find the probability of transmission

$$t \equiv |B|^2$$

To make the algebra easy, **you can assume that** $E = \frac{4}{3}V_0$, $V_0 = \frac{3}{8m}(\frac{2\pi\hbar}{a})^2$.

Show that $r + t = 1$.

(d) Interpret your results, and also discuss the limiting cases:

- $V_0 = 0$,
- $E \gg V_0$,
- and $E = V_0$.