What is the largest possible value of W/Q_{in} for a car engine? ($T_H \approx 600$ K, $T_C \approx 300$ K)
Concepts to keep in mind:

Missing
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- First Law of Thermodynamics
- Second Law of Thermodyanmics
- $\Delta S = Q/T$ if heat flow doesn't change T

Solution We can start with the First Law:

$$Q_{\text{in}} = Q_{\text{out}} + W \tag{1}$$

where the three variables are defined to be positive. This is just energy conservation, the energy into the engine must be equal to the energy out of the engine. We can manipulate this to find the efficiency in terms of Q_{in} and Q_{out}:

$$\text{efficiency} = \frac{W}{Q_{\text{in}}} \tag{2}$$

$$= \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} \tag{3}$$

$$= 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} \tag{4}$$

This means that if we can reduce the amount of energy going into heating our environment, then we can be more efficient. So the question is, how far can we reduce Q_{out}?

Next we can write down the Second Law:

$$\Delta S_H + \Delta S_C \geq 0 \tag{5}$$

where I make use of the fact that the work does not change the entropy of the system or surroundings. But to make any use of this, we need to have an expression for these changes of entropy.

$$\Delta S_H = -\frac{Q_{\text{in}}}{T_H} \tag{6}$$

$$\Delta S_C = \frac{Q_{\text{out}}}{T_C} \tag{7}$$

where the sign of the change comes from whether the reservoir is gaining or losing energy by heating the engine. So let's put these things together with the Second Law:

$$\frac{Q_{\text{out}}}{T_C} - \frac{Q_{\text{in}}}{T_H} \geq 0 \tag{8}$$

We could immediately start manipulating this inequality, but we could also recognize that the smallest possible value for Q_{out} (for a fixed Q_{in}) will be when this is an *equality*, which will give us the greatest possible efficiency. So let's just go with that, and remember that we're talking about the best possible scenario.

$$\frac{Q_{\text{out}}}{T_C} - \frac{Q_{\text{in}}}{T_H} = 0 \tag{9}$$

$$\frac{Q_{\text{out}}}{T_C} = \frac{Q_{\text{in}}}{T_H} \tag{10}$$

$$\frac{Q_{\text{out}}}{Q_{\text{in}}} = \frac{T_C}{T_H} \tag{11}$$

All right, now we have something we can plug into the efficiency.

$$\text{best possible efficiency} = 1 - \frac{T_C}{T_H} \tag{12}$$

This is also known as the Carnot efficiency. It also tells us that our gasoline engine can't possibly be more than 50% efficient because the hot temperature is only twice the cold temperature.

This is a fundamental limit based on the Second Law for the efficiency of any heat engine. In essence it comes from the need for entropy to increase, which is a challenge because extracting energy from your fuel via a heating process inherently *decreases the entropy* of the hot reservoir. So the entropy of something else must increase somewhere else.

If you want to make an engine that is more efficient than this, you need to either increase the temperature on the hot side, or decrease the temperature on the cold side.