In Heat capacity of N_2 we looked at the heat capacity of H_2 and N_2 molecules in a 10 cm cubic box. The discrete energy values for hydrogen were:

1. Translational K.E. in one direction: $\{1 \times 10^{-39}\ \text{J}, 4 \times 10^{-39}\ \text{J}, 9 \times 10^{-39}\ \text{J}, \ldots\}$
2. Rotational K.E.: $\{0, 4 \times 10^{-21}\ \text{J}, 4 \times 10^{-21}\ \text{J}, 4 \times 10^{-21}\ \text{J}, 12 \times 10^{-21}\ \text{J}, \ldots\}$
3. Vibrational energy: $\left\{ \underbrace{4.4 \times 10^{-20}\ \text{J}}_{\text{"zero point energy"}}, 13.2 \times 10^{-20}\ \text{J}, 22 \times 10^{-20}\ \text{J}, \ldots \right\}$

and you found that in the range of temperatures at which N_2 is a gas, the heat capacity (at constant volume) $\frac{dU}{dT}$ per molecule went from $\frac{5}{2}k_B$ at room temperature to $\frac{7}{2}k_B$ at temperatures well above 500 K. In contrast, H_2 had two steps, starting at $\frac{3}{2}k_B$ with the rotations unactivated.

Vibrations

N_2 $\{2.5 \times 10^{-20}\ \text{J}, 7.5 \times 10^{-20}\ \text{J}, 12.5 \times 10^{-20}\ \text{J}, \ldots\}$

H_2 $\{3.5 \times 10^{-20}\ \text{J}, 8 \times 10^{-20}\ \text{J}, 17.5 \times 10^{-20}\ \text{J}, \ldots\}$

Rotations

N_2 $\{0.3 \times 10^{-22}\ \text{J}, 0.6 \times 10^{-22}\ \text{J}, 1.8 \times 10^{-22}\ \text{J}, \ldots\}$

H_2 $\{4 \times 10^{-22}\ \text{J}, 8 \times 10^{-22}\ \text{J}, 24 \times 10^{-22}\ \text{J}, \ldots\}$

Extra material Note about quantum statistics. To find $\frac{dU}{dT}$ rigorously, we need to know the ladder of all allowed quantum energies,

$$\{E_0, E_1, E_2, E_3, \cdots\} \tag{1}$$

and the probability of finding a molecule having a particular energy

$$P_n = \frac{e^{-\frac{E_n}{k_B T}}}{\sum_{i=0}^{\infty} e^{-\frac{E_i}{k_B T}}} \tag{2}$$

What aspects of everyday life are affected by energy quantization?

- Heat capacity of matter
- Flourescent lights
- LED lights
- Thermal radiation (i.e. any other lights, including the sun)
- Solid state electronics

- Chemical reactions
- Photosynthesis and your eyes
- Life itself

It is very rewarding to calculate a list of allowed quantum energies for specific physical systems.

To go deeper into quantum mechanics we must wrestle with two strange ideas:

1. Matter is both a wave and a particle
2. Light is both a wave and a particle

1 Waves

PDE for a light wave traveling in x direction

$$\frac{\partial^2 E_y}{\partial t^2} = (nc)^2 \frac{\partial^2 E_y}{\partial x^2} \tag{3}$$

$$\frac{\partial^2 E_z}{\partial t^2} = (nc)^2 \frac{\partial^2 E_z}{\partial x^2} \tag{4}$$

where E_x and E_y are compontents of the electric field $\vec{E}$, n is the index of refraction of the material, c is the speed of light in vacuum, and nc is the speed of light in the material.

PDE describing a matter wave traveling in x direction (non-relativistic)

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi \tag{5}$$

where ψ is the probability amplitude, $\hbar = \frac{h}{2\pi}$ is Plank's constant, $i^2 = -1$, m is the mass of the particle, and $V(x)$ is the potential energy as a function of position.

A more mundane PDE A wave on a string (good for warming up):

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \tag{6}$$

where y is the displacement of the string from equilibrium, and $v = \sqrt{\frac{LT}{m}}$ is the speed of waves on the string, where m is the string's mass, L is its length, and T is the tension in the string.

Dimension checking When you encounter new equations, particularly new *kinds* of equations like partial differential equations, it's worth checking that the dimensions work out. I'll talk through this, but the key is to recognize that the dimensions of $\frac{\partial^2}{\partial x^2}$ are inverse length squared, and the dimensions of $\frac{\partial^2}{\partial t^2}$ are inverse time squared, etc. Which allow us to confirm that each of these wave equations have the correct dimensions.

1.1 Example 1

A slack line, tied between two trees, and you can walk on it.

Missing

/var/www/paradigms_media_2/media/activity_media/s

Figure 1: A slack line (source)

You have learned in intro physics that

$$f_1 = \frac{v}{2L} \qquad\qquad v = \sqrt{\frac{T}{m/L}} \tag{7}$$

for the fundamental frequency of a string under tension, and the higher harmonics have frequencies that are an integer times the fundamental, i.e. $f_n = nf_1$.

For a 20 meter long slack line $\frac{m}{L} = 0.05$ kg/m and we can crank up the tension to $T = 2000$ N. This gives us that

$$v = \sqrt{\frac{2\times 10^3 \text{ kg m s}^{-2}}{0.05 \text{ kg/m}}} \tag{8}$$

$$= \sqrt{4\times 10^4 \text{ m}^2/\text{s}^2} \tag{9}$$

$$= 2\times 10^2 \text{ m/s} \tag{10}$$

$$= 200 \text{ m/s} \tag{11}$$

$$f_1 = \frac{200 \text{ m/s}}{40 \text{ m}} = 5 \text{ s}^{-1} = 5 \text{ Hz} = 5 \text{ cycles per sec.} \tag{12}$$

$$f = \{5 \text{ Hz}, 10 \text{ Hz}, 15 \text{ Hz}, \cdots\} \tag{13}$$

FIXME add frequency level diagram to show the allowed values.

Strings under tension have other behavior, too. More than “pure sinusoidal standing waves.” See e.g. this animated solution to the wave equation.