

One of the pathways for hydrogen fusion involves the following reaction:



1. How much kinetic energy does a deuterium nucleus need to fuse with a tritium nucleus? *Note: for a coarse-grained approximation, you may assume that the tritium is fixed in position and the deuterium speeds toward it.*

**Solution** We first need to find out how big the nuclei are using

$$r_{\text{nucleus}} \approx [1.2 \text{ fm}] A^{\frac{1}{3}} \quad (2)$$

$$= [1.2 \text{ fm}] 2^{\frac{1}{3}} \text{ and } [1.2 \text{ fm}] 3^{\frac{1}{3}} \quad (3)$$

$$= 1.5 \text{ fm and } 1.7 \text{ fm} \quad (4)$$

Taken together, we'll say that the nuclei need to get within 3 fm to fuse.

We use the Coulomb energy:

$$U_{\text{elec}} = k_C \frac{q_1 q_2}{r} \quad (5)$$

$$= \left[ 9 \times 10^9 \frac{\text{J} \cdot \text{m}}{\text{C}^2} \right] \frac{[1.6 \times 10^{-19} \text{ C}]^2}{3 \times 10^{-15} \text{ m}} \quad (6)$$

$$\approx 8 \times 10^{-14} \text{ J} \quad (7)$$

2. If the deuterium is in a gas, what temperature should the gas be so that most deuterium has enough kinetic energy for fusion?

**Solution** Equipartition theorem tells us that the kinetic energy of each atom is  $\frac{3}{2}k_B T$ . Since we have two atoms involved, we can have twice that energy available for a collision ( $3k_B T$ ) that. So we can find the temperature needed by setting this equal to the energy needed to touch the two nuclei

$$T = \frac{U_{\text{elec}}}{3k_B} \quad (8)$$

$$= \frac{8 \times 10^{-14} \text{ J}}{3 \cdot 1.4 \times 10^{-23} \text{ J/K}} \quad (9)$$

$$\approx 2 \times 10^9 \text{ K} \quad (10)$$

This is rather hot, far hotter than the surface of the sun. It's also why we don't yet have practical fusion reactors.

3. **Extra** Based on the masses of deuterium ( ${}^2\text{H}$ ) and tritium ( ${}^3\text{H}$ ), and given that one neutron will be produced, how much energy will be generated when they fuse?

| isotope          | mass      |
|------------------|-----------|
| ${}^2\text{H}$   | 2.0141 u  |
| ${}^3\text{H}$   | 3.01605 u |
| ${}^4\text{He}$  | 4.0260 u  |
| ${}^1_0\text{n}$ | 1.0087 u  |

**Solution** The change in mass is

$$\Delta m = 2.0141 \text{ u} + 3.01605 \text{ u} - 4.0260 \text{ u} - 1.0087 \text{ u} \quad (11)$$

$$= -0.00455 \text{ u} \quad (12)$$

$$= -7.6 \times 10^{-30} \text{ kg} \quad (13)$$

It's good news that the mass decreased, since otherwise we wouldn't get any energy out of the reaction.

$$\text{energy generated} = |\Delta m|c^2 \quad (14)$$

$$= [7.6 \times 10^{-30} \text{ kg}][3 \times 10^8 \text{ m/s}]^2 \quad (15)$$

$$= 7 \times 10^{-13} \text{ J} \quad (16)$$

This is a lot less energy than we got out of splitting one uranium atom.

4. **Extra** What mass of deuterium + tritium would need to be fused to provide the electrical power the United States uses in a day?<sup>1</sup>

**Solution**

$$\text{mass needed per day} = \frac{[10^{12} \text{ J/s}]}{[7 \times 10^{-13} \text{ J/pair of atoms}]} [8 \times 10^{-27} \text{ kg/pair of atoms}] [9 \times 10^4 \text{ s/day}] \quad (17)$$

$$\approx 10^3 \text{ kg/day} \quad (18)$$

So we do need a lot more tritium and deuterium per day than we do uranium to power the United States. But it's still  $4 \times 10^6$  times less mass than the amount of coal we need per day. Plus it wouldn't produce any carbon dioxide, which is a major plus.

<sup>1</sup>The electrical power use of the US is about  $10^{12}$  J/s.