

You may want to start by reviewing the introduction to Plotting with matplotlib. In this activity, I'll give you a variety of interesting functions to plot based on special relativity.

**Task 1:**  $\gamma$  Let's start by plotting  $\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$  as a function of  $\beta$ , which is equal to  $v/c$ , where  $c$  is the speed of light.

### Your task

1. Plot  $\gamma$  vs  $\beta$ . Keep in mind  $\beta$  should be on the horizontal axis.
2. Label your two axes.
3. You may be aware that in the limiting case of small speeds (i.e. the classical limit),  $\gamma \approx 1 + \frac{1}{2}\beta^2$ . This expression tells us that the famous expression for relativistic energy becomes

$$E = \gamma mc^2 \quad (1)$$

$$\approx \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) mc^2 \quad (2)$$

$$= mc^2 + \frac{1}{2}mv^2 \quad (3)$$

Which is to say, the total energy in the classical limit is the mass energy plus the classical kinetic energy.

Plot this classical approximation for  $\gamma$  on the same plot, and see how it compares with the exact expression.

4. Add a legend to distinguish your two curves.

### Lorentz transformation Start a new Python file for this task!

We will next look at the Lorentz transformation (considering just the two coordinates  $x$  and  $t$ ):

$$t' = \gamma \left( t - \frac{vx}{c^2} \right) \quad (4)$$

$$x' = \gamma(x - vt) \quad (5)$$

Here  $x$  and  $t$  are the coordinates in the lab frame, and  $t'$  and  $x'$  are the coordinates in a frame moving with velocity  $v$  in the  $\hat{x}$  direction.

**Your task** You can remain on the ground, while I travel in a spaceship at speed  $v$ . Or if you like, on a train.

1. Start by labeling your axes with the vertical axis as  $t$  and the horizontal axis as  $x$ . These will be your lab coordinates.

2. Define a speed  $v$  in your code (select a value), and from it compute  $\gamma$ . I suggest using units in which  $c = 1$  for this work, but you are not required to do so. Then plot a curve that has  $x' = 0$ , and a separate curve that has  $t' = 0$ . These two curves would represent what  $I$  think are the time and position axes, respectively. Play with  $v$  and see how these axes change.
3. Now let's imagine my spaceship has a length  $d$  in its rest frame, and I'm sitting at the back of it. You'll have to define a variable holding  $d$ , and pick a value for it. Plot a curve  $x' = d$ , which represents the location of the front of my ship. Make my ship go fast enough that you can observe length contraction.
4. Finally, let's have me have the hiccups and hiccup at times  $0, T, 2T, 3T$ , etc. Draw curves representing the times that I hiccup, which is when  $t' = T$ , etc. See if you can observe time dilation.