Consider the Lie algebra $\mathfrak{so}(5,1)$ corresponding to the orthogonal group $SO(5,1)$.

1. What size are the matrices $X \in \mathfrak{so}(5,1)$?
2. What is the dimension of the vector space $\mathfrak{so}(5,1)$? That is, how many independent rotations $M(\alpha) \in SO(5,1)$ are there? Equivalently, how many independent generators $M'(0) \in \mathfrak{so}(5,1)$ are there?
3. The Killing form B is an inner product on $\mathfrak{so}(5,1)$ (although not necessarily positive definite). What size is the matrix representation of B?
4. What is the signature of B? That is, how many rotations are there, and how many boosts?

Solution

1. There are six coordinates acted on by $SO(5,1)$ (also by $\mathfrak{so}(5,1)$), so the matrices in both $SO(5,1)$ and $\mathfrak{so}(5,1)$ are 6×6.
2. There is one generalized rotation for each plane. In six dimensions, there are $\binom{6}{2} = 15$ independent planes, so $\mathfrak{so}(5,1)$ is a 15-dimensional vector space.
3. The Killing form is therefore an inner product ("dot product") on a 15-dimensional vector space, so its matrix representation must be 15×15.
4. There are five ordinary (spacelike) coordinates, and one timelike coordinate. So there are $\binom{5}{2} = 10$ rotations, and 5 boosts − one for each independent Lorentz transformation.