Vector Differential: Polar Coordinates:

Find the general form for $d\vec{r}$ in polar coordinates by determining $d\vec{r}$ along the specific paths in the figure below.

- Path 1:

  $d\vec{r} =$

- Path 2:

  $d\vec{r} =$

If both coordinates are allowed to change simultaneously, by an infinitesimal amount, we could write this $d\vec{r}$ for any path as:

$d\vec{r} =$

This is the general line element in polar coordinates.

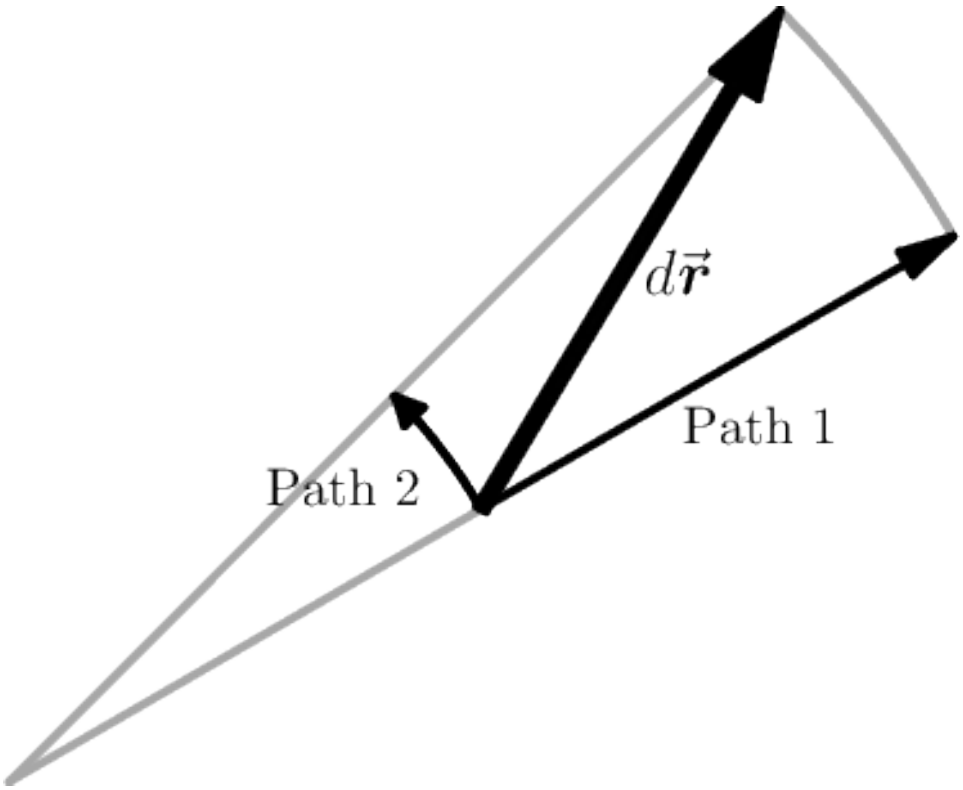

Figure 1: Figure 1: $d\vec{r}$ in polar coordinates

Solution

- Path 1:

$$d\vec{r} = dr\,\hat{r}$$

- Path 2:

$$d\vec{r} = r d\phi\,\hat{\phi}$$

- In general:

$$d\vec{r} = dr\,\hat{r} + r d\phi\,\hat{\phi}$$