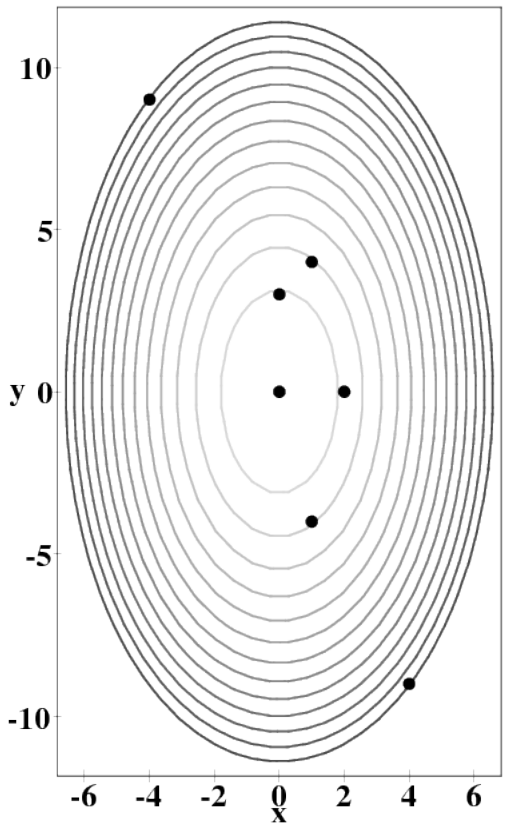
Suppose you are standing on a hill. You have a topographic map, which uses rectangular coordinates (x, y) measured in miles. Your global positioning system says your present location is at one of the following points (pick one):



A: $(1, 4)$ **B:** $(4, -9)$ **C:** $(-4, 9)$ **D:** $(1, -4)$ **E:** $(2, 0)$ **F:** $(0, 3)$

Your guidebook tells you that the height h of the hill in feet above sea level is given by

$$h = a - bx^2 - cy^2$$

where $a = 5000\text{ft}$, $b = 30\,\frac{\text{ft}}{\text{mi}^2}$, and $c = 10\,\frac{\text{ft}}{\text{mi}^2}$.

- Starting at your present location, in what map direction (2-d unit vector) do you need to go in order to climb the hill as steeply as possible?

Solution Using $(4, -9)$, $\frac{\vec{\nabla} h}{|\vec{\nabla} h|} = \frac{1}{300}(-240\,\hat{\boldsymbol{x}} + 180\,\hat{\boldsymbol{y}})$

Draw this vector on your topographic map.

- How steep is the hill if you start at your present location and go in this compass direction?

> **Solution** The steepness is given by the magnitude of the graident. At point C, the magnitude is 300 feet per mile. Be careful with the units! There is no reason that the units for the range of the function need to be related to the units of the domain.

Draw a picture which shows the slope of the hill at your present location.

- In what direction in space (3-d vector) would you actually be moving if you started at your present location and walked in the map direction you found above?

> **Solution** $-\frac{4}{5}\,\hat{\boldsymbol{x}} + \frac{3}{5}\,\hat{\boldsymbol{y}} + 300\,\hat{\boldsymbol{z}}$

To simplify the computation, your answer does ***not*** *need to be a unit vector.*