

Kepler's 2nd Law follows directly from conservation of angular momentum.

First, remember that the magnitude of the vector cross product between two vectors is the area of the parallelogram spanned by the two vectors:

$$|\vec{a} \times \vec{b}| = \text{Area of Parallelogram}$$

In its orbit in a small amount of time dt , the reduced mass sweeps out a sector that is approximately a triangle with area:

$$dA = \frac{1}{2} |\vec{r} \times \vec{v} dt|$$

where $\vec{r}$ is the position vector and $\vec{v}$ is the velocity of the reduced mass. $\vec{v} dt$ is the small arc length that the reduced mass travels in time dt .

I can divide both sides by dt to determine the rate that area is swept out in the orbital plane.

$$\frac{dA}{dt} = \frac{1}{2} |\vec{r} \times \vec{v}|$$

I can write the right hand side in terms of the momentum:

$$\frac{dA}{dt} = \frac{1}{2m} |\vec{r} \times \vec{p}|$$

and recognize that the cross product is the angular momentum:

$$\frac{dA}{dt} = \frac{|\vec{L}|}{2m}$$

If the angular momentum is conserved, that the rate area is swept out by the reduced mass is constant.

$$\frac{dA}{dt} = \text{constant}$$