

Electrostatic Potential from Two Charges

- Find a formula for the electrostatic potential $V(\vec{r})$ that is valid everywhere in space for:
 - Two charges $+Q$ and $+Q$ placed on the z -axis at $z' = D$ and $z' = -D$.
 - Two charges $+Q$ and $-Q$ placed on the z -axis at $z' = D$ and $z' = -D$, respectively.

Solution

- Two charges $+Q$ and $+Q$:

$$V(x, y, z) = kQ \left(\frac{1}{\sqrt{x^2 + y^2 + (z - D)^2}} + \frac{1}{\sqrt{x^2 + y^2 + (z + D)^2}} \right)$$

- Two charges $+Q$ and $-Q$:

$$V(x, y, z) = kQ \left(\frac{1}{\sqrt{x^2 + y^2 + (z - D)^2}} - \frac{1}{\sqrt{x^2 + y^2 + (z + D)^2}} \right)$$

- Simplify your formulas for the special cases of:
 - the x -axis

Solution

- * Two charges $+Q$ and $+Q$:

$$V(x, 0, 0) = \frac{2kQ}{\sqrt{x^2 + D^2}}$$

- * Two charges $+Q$ and $-Q$:

$$V(x, 0, 0) = 0$$

- the z -axis

Solution

- * Two charges $+Q$ and $+Q$:

$$V(0, 0, z) = kQ \left(\frac{1}{\sqrt{(z - D)^2}} + \frac{1}{\sqrt{(z + D)^2}} \right) \quad (1)$$

$$= kQ \left(\frac{1}{|z - D|} + \frac{1}{|z + D|} \right) \quad (2)$$

- * Two charges $+Q$ and $-Q$:

$$V(0, 0, z) = kQ \left(\frac{1}{\sqrt{(z - D)^2}} - \frac{1}{\sqrt{(z + D)^2}} \right) \quad (3)$$

$$= kQ \left(\frac{1}{|z - D|} - \frac{1}{|z + D|} \right) \quad (4)$$

- Discuss the relationship between the symmetries of the physical situations and the symmetries of the functions in these special cases.

Solution

- Two charges $+Q$ and $+Q$: This charge distribution is symmetric across both the x - and z -axes. The function

$$V(x, 0, 0) = \frac{2kQ}{\sqrt{x^2 + D^2}}$$

is symmetric under the interchange $x \rightarrow -x$; it is an even function of x . And the function

$$V(0, 0, z) = kQ \left(\frac{1}{|z - D|} + \frac{1}{|z + D|} \right) \quad (5)$$

is symmetric under the interchange $z \rightarrow -z$; it is an even function of z .

- Two charges $+Q$ and $-Q$: This charge distribution is symmetric across the z -axis, but antisymmetric across the x -axis. The function

$$V(x, 0, 0) = 0$$

is both symmetric and antisymmetric under the interchange $x \rightarrow -x$; the function 0 is both even and odd. And the function

$$V(0, 0, z) = kQ \left(\frac{1}{|z - D|} - \frac{1}{|z + D|} \right) \quad (6)$$

is antisymmetric under the interchange of $z \rightarrow -z$; it is an odd function of z .