

$$|v\rangle \doteq \frac{1}{\sqrt{74}} \begin{pmatrix} 7 \\ 5i \end{pmatrix}$$

$$|+\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |-\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

1. You can expand a two-component vector with complex entries in terms of the standard basis,  $|+\rangle$  and  $|-\rangle$ . Find the coefficients  $v_+$  and  $v_-$  for the vector  $|v\rangle$  in the expression

$$|v\rangle = v_+ |+\rangle + v_- |-\rangle$$

### Solution

$$v_+ = \langle +|v\rangle \tag{1}$$

$$= \begin{pmatrix} 1 & 0 \end{pmatrix} \frac{1}{\sqrt{74}} \begin{pmatrix} 7 \\ 5i \end{pmatrix} \tag{2}$$

$$= \frac{7}{\sqrt{74}} \tag{3}$$

$$v_- = \langle -|v\rangle \tag{4}$$

$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \frac{1}{\sqrt{74}} \begin{pmatrix} 7 \\ 5i \end{pmatrix} \tag{5}$$

$$= \frac{5i}{\sqrt{74}} \tag{6}$$

2. Try a different basis:

$$|+\rangle_y \doteq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \quad |-\rangle_y \doteq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Find:

$$|v\rangle = v_{+y} |+\rangle_y + v_{-y} |-\rangle_y$$

### Solution

$$v_{+y} = {}_y\langle +|v\rangle \tag{7}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \end{pmatrix} \frac{1}{\sqrt{74}} \begin{pmatrix} 7 \\ 5i \end{pmatrix} \tag{8}$$

$$= \frac{12}{\sqrt{128}} \tag{9}$$

$$v_{-y} = {}_y \langle -|v \rangle \quad (10)$$

$$= \frac{1}{\sqrt{2}} (0 \quad i) \frac{1}{\sqrt{74}} \begin{pmatrix} 7 \\ 5i \end{pmatrix} \quad (11)$$

$$= \frac{2}{\sqrt{128}} \quad (12)$$