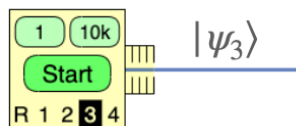


Finding the Unknown States Leaving an Oven (Spin- $\frac{1}{2}$)

1. Launch the Spins Laboratory simulation and choose Unknown # 3 on the oven. This causes the atoms to come out of the oven in a definite quantum state (instead of a random state), which we call $|\psi_3\rangle$.



2. Assume that we want to write the unknown state vectors in terms of the $|\pm\rangle$ basis, *i.e.* $|\psi_3\rangle = a|+\rangle + be^{i\gamma}|-\rangle$, where a and b are real. We thus must use the data to find the values of a , b and γ . Measure the six probabilities $|\langle \pm | \psi_3 \rangle|^2$, where $|\pm\rangle_n$ corresponds to the spin states (“spin up” and “spin down”) along the three axes $n = x, y$ or z . Fill in the table for $|\psi_3\rangle$ on the worksheet.
3. Using the probabilities determined from your experiments, calculate Unknown #3 ($|\psi_3\rangle$) and write it in the $|\pm\rangle$ basis.
4. Using the SPINS program, design and run a simulated experiment to verify your calculated state (Hint: recall the general spin- $\frac{1}{2}$ state vector can be written as $|+\rangle_n = \cos \frac{\theta}{2} |+\rangle + \sin \frac{\theta}{2} e^{i\phi} |-\rangle$).
5. Repeat this exercise for Unknown # 4 ($|\psi_4\rangle$).

 Unknown $|\psi_3\rangle$

Probabilities	Spin Component		
Result	S_x	S_y	S_z
$\mathcal{P}(\frac{\hbar}{2})$			
$\mathcal{P}(-\frac{\hbar}{2})$			

Solution Probabilities:

$1/2, 97/100, 1/3$

$1/2, 3/100, 2/3,$

$$|\psi_3\rangle = \sqrt{\frac{1}{3}}|+\rangle + \sqrt{\frac{2}{3}}e^{i\pi/2}|-\rangle$$

Explanation

I can write the state generically as:

$$|\psi_4\rangle = a|+\rangle_z + be^{i\phi}|-\rangle_z$$

where a and b are real.

The probabilities of the S_z component of spin tell me a and b :

$$\begin{aligned}\mathcal{P}(S_z = \hbar/2) &= \frac{1}{3} \\ |{}_z\langle +|\psi_3\rangle|^2 &= |a|^2 \\ \rightarrow a &= \frac{1}{\sqrt{3}}\end{aligned}$$

$$\begin{aligned}\mathcal{P}(S_z = -\hbar/2) &= \frac{2}{3} \\ |{}_z\langle -|\psi_3\rangle|^2 &= |be^{i\phi}|^2 = |b|^2 \\ \rightarrow b &= \sqrt{\frac{2}{3}}\end{aligned}$$

To find the relative phase, I'll look at the probabilities for the S_x and S_y components:

$$\begin{aligned}\mathcal{P}(S_x = \hbar/2) &= \frac{1}{2} \\ |{}_x\langle +|\psi_3\rangle|^2 &= \left| \left(\frac{1}{\sqrt{2}}{}_z\langle +| + \frac{1}{\sqrt{2}}{}_z\langle -| \right) \left(\frac{1}{\sqrt{3}}|+\rangle_z + \sqrt{\frac{2}{3}}e^{i\phi}|-\rangle_z \right) \right|^2 \\ &= \left| \frac{1}{\sqrt{6}} + \sqrt{\frac{2}{6}}e^{i\phi} \right|^2 \\ &= \frac{1}{6} \left(1 + \sqrt{2}e^{i\phi} \right) \left(1 + \sqrt{2}e^{-i\phi} \right) \\ &= \frac{1}{6} \left(1 + \sqrt{2}(e^{i\phi} + e^{-i\phi}) + 2 \right) \\ &= \frac{1}{6} \left(3 + 2\sqrt{2}\cos\phi \right) \\ \frac{1}{2} &= \frac{1}{2} + \frac{\sqrt{2}}{3}\cos\phi \\ 0 &= \cos\phi \\ \phi &= \pm\frac{\pi}{2}\end{aligned}$$

This gives me two possible phase angles. Now try the an S_y probability:

$$\mathcal{P}(S_y = \hbar/2) = \frac{97}{100}$$

$$|_y \langle + | \psi_3 \rangle|^2 = \left| \left(\frac{1}{\sqrt{2}} z \langle + | - \frac{i}{\sqrt{2}} z \langle - | \right) \left(\frac{1}{\sqrt{3}} |+\rangle_z + \sqrt{\frac{2}{3}} e^{i\phi} |+\rangle_z \right) \right|^2$$

$$= \left| \frac{1}{\sqrt{6}} + i \sqrt{\frac{2}{6}} e^{i\phi} \right|^2$$

$$= \frac{1}{6} \left(1 + i\sqrt{2}e^{i\phi} \right) \left(1 - i\sqrt{2}e^{-i\phi} \right)$$

$$= \frac{1}{6} \left(1 + i\sqrt{2}(e^{i\phi} - e^{-i\phi}) + 2 \right)$$

$$= \frac{1}{6} \left(3 + 2\sqrt{2} \sin \phi \right)$$

$$\frac{97}{100} = \frac{1}{2} + \frac{\sqrt{2}}{3} \sin \phi$$

$$\frac{47}{100} = \frac{\sqrt{2}}{3} \sin \phi$$

$$\sin \phi = \frac{(3)(47)}{100\sqrt{2}}$$

$$\sin \phi \approx 1$$

$$\phi = +\frac{\pi}{2}$$

Unknown $|\psi_4\rangle$

Probabilities	Spin Component		
Result	S_x	S_y	S_z
$\mathcal{P}(\frac{\hbar}{2})$			
$\mathcal{P}(-\frac{\hbar}{2})$			

Solution Probabilities:

1/2, 14/15, 1/4

1/2, 1/15, 3/4,

$$|\psi_4\rangle = \frac{1}{2} |+\rangle + \frac{\sqrt{3}}{2} e^{i\pi/2} |-\rangle$$