

Eigenvalues and Eigenvectors

Each group will be assigned one of the following matrices.

$$A_1 \doteq \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad A_2 \doteq \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad A_3 \doteq \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A_4 \doteq \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \quad A_5 \doteq \begin{pmatrix} 3 & -i \\ i & 3 \end{pmatrix} \quad A_6 \doteq \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad A_7 \doteq \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$$

$$A_8 \doteq \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad A_9 \doteq \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S_x \doteq \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad S_y \doteq \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad S_z \doteq \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For your matrix:

1. Find the eigenvalues.
2. Find the (unnormalized) eigenvectors.
3. Describe what this transformation does.
4. Normalize your eigenvectors. (Quantum eigenstates are represented by normalized eigenvectors.)

If you finish early, try another matrix with a different structure, *i.e.* real vs. complex entries, diagonal vs. non-diagonal, 2×2 vs. 3×3 , with vs. without explicit dimensions.