As we learned last week, **heat capacity** is the ammount of energy required to reaise the temperature of an object by a small amount.

$$đQ = CdT \qquad\qquad \textbf{At constant what?} \tag{1}$$

If we hold the volume constant, then we can see from the first law that

$$dU = đQ - pdV$$

since $dV = 0$ for a constant-volume process,

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

But we didn't measure C_V in our calorimetry lab, since we didn't hold the volume of the water constant. Instead we measured C_p since we held the pressure constant. But what derivative is that? To distinguish between different sorts of heat capacities, we need to specify the sort of path used. So, for instance, we could write

$$\begin{align}
đQ &= TdS \tag{2}\\
đQ &= C_\alpha dT + ?d\alpha \tag{3}\\
TdS &= C_\alpha dT + ?d\alpha \tag{4}\\
dS &= \frac{C_\alpha}{T}dT + \frac{?}{T}d\alpha \tag{5}\\
C_\alpha &= T\left(\frac{\partial S}{\partial T}\right)_\alpha \tag{6}
\end{align}$$

This may look like an overly-tricky derivative, so let's go throught the first law and check that we got it right in a few cases. I'll do the C_V case. We already know that

$$\begin{align}
dU &= đQ - pdV \tag{7}\\
C_V &= \left(\frac{\partial U}{\partial T}\right)_V \tag{8}\\
&= \left(\frac{\partial U}{\partial S}\right)_V \left(\frac{\partial S}{\partial T}\right)_V \tag{9}\\
&= T\left(\frac{\partial S}{\partial T}\right)_V \tag{10}
\end{align}$$

Where the second step just uses the ordinary chain rule.