

Using the applet at Inner Products of Functions, convince yourself of the following:
For integer m and m'

$$\int_0^L \sin \frac{2\pi m x}{L} \sin \frac{2\pi m' x}{L} dx = \begin{cases} \frac{L}{2} & \text{if } m = m' \\ 0 & \text{if } m \neq m' \end{cases}$$

$$\int_0^L \sin \frac{2\pi m x}{L} \cos \frac{2\pi m' x}{L} dx = \begin{cases} 0 & \text{if } m = m' \\ 0 & \text{if } m \neq m' \end{cases}$$

$$\int_0^L \cos \frac{2\pi m x}{L} \cos \frac{2\pi m' x}{L} dx = \begin{cases} \frac{L}{2} & \text{if } m = m' \neq 0 \\ L & \text{if } m = m' = 0 \\ 0 & \text{if } m \neq m' \end{cases}$$